

Non-Hermitian \mathcal{PT} -symmetric relativistic quantum theory in an intensive magnetic field

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Abstract

We develop relativistic non-Hermitian quantum theory and its application to neutrino physics in a strong magnetic field. It is well known, that one of the fundamental postulates of quantum theory is the requirement of Hermiticity of physical parameters. This condition not only guarantees the reality of the eigenvalues of Hamiltonian operators, but also implies the preservation of the probabilities of the considered quantum processes. However as it was shown relatively recently (Bender, Boettcher 1998), Hermiticity is a sufficient but it is not a necessary condition. It turned out that among non-Hermitian Hamiltonians it is possible to allocate a number of such which have real energy spectra and can ensure the development of systems over time with preserving unitarity. This type of Hamiltonians includes so-called parity-time (\mathcal{PT}) symmetric models which is already used in various fields of modern physics. The most developed in this respect are models, which used in the field of \mathcal{PT} -symmetric optics, where for several years produced not only theoretical but experimental studies.

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1 Introduction

It is well known the Nobel Prize in Physics was awarded 2015 jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass". This discovery has completely changed our understanding of the innermost properties of matter and showed that Standard Model (SM) cannot be the comprehensive theory of the fundamental constituents of the Universe. Obviously, that of past

successes of the SM is so high that new models which is designed for modifying SM, should contain practically all basic principles lying in the basis of already existing theory. It is important to note that the generalization of the notion of Hermiticity has a strict quantum mechanical justification. Indeed it is known one of the fundamental postulates of quantum theory is the requirement of Hermiticity of physical parameters. This condition not only guarantees the reality of the eigenvalues of Hamilton operators, but also implies the preservation in time of the probabilities of the considered quantum processes. However, as was shown relatively recently [1], Hermiticity is a sufficient but it is not a necessary condition. It turned out that among non-Hermitian Hamiltonians it is possible to allocate a wide number having real energy spectra and providing the development of systems over time with preserving unitarity.

Now it is well-known fact, that the reality of the spectrum in models with a non-Hermitian Hamiltonian is a consequence of \mathcal{PT} -invariance of the theory, i.e. a combination of spatial and temporary parity of the total Hamiltonian: $[H, \mathcal{PT}]\psi = 0$. When the \mathcal{PT} symmetry is unbroken, the spectrum of the quantum theory will be real. This surprising results explain the growing interest in this problem which was initiated by Bender and Boettcher's observation [1]. For the past a few years has been studied a lot of new non-Hermitian \mathcal{PT} -invariant systems (see, for example [2] - [8]).

The algebraic non-Hermitian \mathcal{PT} -symmetric γ_5 -extension of the Dirac equation was first studied in [2] and further was developed in the works [3]-[5]. However in the geometrical approach to the construction of Quantum Field Theory (QFT) with fundamental mass which was developed by V.G.Kadyshevsky, equation for motion fermion with γ_5 -mass extension [9] was obtained yet in seventies years of the last century (see also [10],[11]). The purpose of the present paper is the continuation of the studying examples of pseudo-Hermitian relativistic Hamiltonians, investigations of which was started by us earlier (see [3]-[5]). In the papers [12],[13] the energy spectra of the fermions was obtained by us as exact solutions of the modified Dirac equation in which taken into account the interaction of anomalous magnetic moment (AMM) of fermions with uniform magnetic field.

On the other hand in 1965 M.A.Markov [14] has proposed hypothesis according to which the mass spectrum of particles should be limited by the Planck mass $m_{Planck} = 10^{19} GeV$. The particles with the limiting mass

$$m \leq m_{Planck} \tag{1}$$

were named by the author Maximons. However, condition (1) initially was purely phenomenological and until recently it has seemed that this restriction can be applied without connection with SM. And really SM is irreproachable scheme for value of mass from zero till infinity. But in the current situation, however, more and more data are accumulated that bear witness in favor of the necessity of revising some physical principles. In particular, this is confirmed by abundant evidence that "dark matter", apparently exists and absorbs a substantial part of the energy density in the Universe.

In the late 1970s, a new radical approach was offered by V. G. Kadyshesky [9] (see also [10],[11]), in which the Markov's idea of the existence of a maximal mass used as new fundamental principle construction of QFT. This principle refutes the affirmation that mass of the elementary particle can have a value in the interval $0 \leq m < \infty$. In the geometrical theory the condition finiteness of the mass spectrum is postulated in the form

$$m \leq \mathcal{M}, \quad (2)$$

where the maximal mass parameter \mathcal{M} , was named by the the *fundamental mass*. This physical parameter is a *new physical constant* along at the speed of light and Planck's constant. The value of \mathcal{M} is considered as a curvature radius of a five dimensional hyperboloid whose surface is a realization of the curved momentum 4-space – the anti de Sitter space. Objects with a mass larger than \mathcal{M} cannot be regarded as elementary particles because no local fields that correspond to them. For a free particle, condition (2) holds automatically on surface of a five dimensional hyperboloid. In the approximation $\mathcal{M} \gg m$ the anti de Sitter geometry goes over into the Minkowski geometry in the four dimensional pseudo-Euclidean space ("flat limit")[9].

Here we are producing our investigation of non-Hermitian systems with γ_5 -mass contribution taking into account AMM of fermions in external magnetic field. In Section 2 we consider restriction of mass in pseudo-Hermitian algebraic theory. We also are studying the spectral and polarization properties of such systems (Section 3). The novelty of our approach is associated with predictions of new phenomena caused by a number of additional terms arising in the non-Hermitian Hamiltonians (Section 4). Intriguing predictions in our papers [12],[13] are connected with non-Hermitian mass extension and associated with the appearance in this the algebraic approach of any new particles. It is important that previously such particles ("exotic particles") was observed only in the framework of the geometric approach to the construction of QFT.

2 Restriction of mass in pseudo-Hermitian algebraic theory

The inequality $m_1 \geq m_2$ in this theory follows from the condition $m^2 = m_1^2 - m_2^2$, which is the basic requirement that defines unbroken symmetry of the Hamiltonian [2]. However, this inequality between m , m_1 and m_2 is not single condition, which links the parameters of γ_5 -extension of mass. In particular we can write the new condition for the physical mass m , which may be more substantial. Indeed, using the simple mathematical theorem, we can obtain inequality in the form [4]

$$m \leq m_1^2/2m_2 = M \quad (3)$$

where the new parameter M restricts change of mass m (the details of this approach one can see at Fig.1).

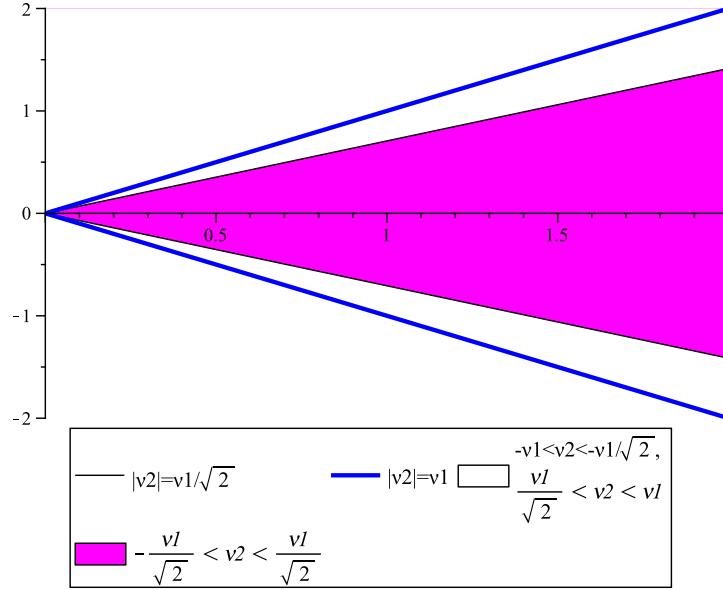


Figure 1: The parametric domain of unbroken \mathcal{PT} -symmetry $m_1^2 \geq m_2^2$ for non-Hermitian Hamiltonian comprises three characteristic sub-domains: the shaded domain II corresponds to standard particles and the neighboring domains I and III correspond to the description of exotic fermions.

Introducing the normalized values $\nu = m/M$, $\nu_1 = m_1/M$, $\nu_2 = m_2/M$

and solving the system of equations $m^2 = m_1^2 - m_2^2$ and (3) we have expressions with two signs for values parameters ν_1 and ν_2 :

$$\nu_1^\mp = \sqrt{2(1 \mp \sqrt{1 - \nu^2})}; \quad \nu_2^\mp = (1 \mp \sqrt{1 - \nu^2}). \quad (4)$$

We recall that we are investigating the issue of the existence of constraints on mass parameters in the given theory. It is shown by us that there is a constraint on the parameter m in the algebraic theory. In this case, there are reasons to believe that a direct relationship exists between M obtained by algebraic way and \mathcal{M} which is consequence the *geometric approach to modified QFT with the fundamental mass* [3],[4].

Let us now consider obtaining the modified Dirac equations for free massive particles using the γ_5 -factorization of the ordinary Klein-Gordon operator. It is interesting that in this case we can use simple way similar to known Dirac's procedure. As he wrote: "...get something like a square root from the equation Klein-Gordon" [15]. And really if we shall not be restricted to only Hermitian operators then we can represent the Klein-Gordon operator in the form of a product of two commuting matrix operators with γ_5 -mass extension (where $\hbar = c = 1$):

$$\left(\partial_\mu^2 + m^2\right) = \left(i\partial_\mu\gamma^\mu - m_1 - \gamma_5 m_2\right)\left(-i\partial_\mu\gamma^\mu - m_1 + \gamma_5 m_2\right), \quad (5)$$

where the physical mass of particles m is expressed through the parameters m_1 and m_2

$$m^2 = m_1^2 - m_2^2. \quad (6)$$

For the function would obey to the equations of Klein-Gordon

$$\left(\partial_\mu^2 + m^2\right)\tilde{\psi}(x, t) = 0 \quad (7)$$

one can demand that it also satisfies to one of equations of the first order

$$\left(i\partial_\mu\gamma^\mu - m_1 - \gamma_5 m_2\right)\tilde{\psi}(x, t) = 0; \quad \left(-i\partial_\mu\gamma^\mu - m_1 + \gamma_5 m_2\right)\tilde{\psi}(x, t) = 0 \quad (8)$$

Equations (8) of course, are less common than (7), and although every solution of one of the equations (8) satisfies to (7), reverse approval has not designated.

It is also obvious that the Hamiltonians, associated with the equations (8), are non-Hermitian (pseudo-Hermitian), because in them the γ_5 -dependent mass components appear ($H \neq H^+$):

$$H = \vec{\alpha} \mathbf{p} + \beta(m_1 + \gamma_5 m_2) = \vec{\alpha} \mathbf{p} + \beta m e^{\gamma_5 \alpha} \quad (9)$$

and

$$H^+ = \vec{\alpha} \mathbf{p} + \beta(m_1 - \gamma_5 m_2) = \vec{\alpha} \mathbf{p} + \beta m e^{-\gamma_5 \alpha}. \quad (10)$$

Here matrices $\alpha_i = \gamma_0 \cdot \gamma_i$, $\beta = \gamma_0$, $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$ and introduced identical replacement of parameters

$$\sinh(\alpha) = m_2/m; \quad \cosh(\alpha) = m_1/m, \quad (11)$$

where parameter α varies from zero to infinity.

It is easy to see from (6) that the mass m , appearing in the equation (7) is real, when the inequality

$$m_1^2 \geq m_2^2, \quad (12)$$

is accomplished[2]. However for variable α which is identical for definitions m_1 , m_2 this condition is automatically accomplished in all region of α changes.

However this area contains descriptions not only pseudo-Hermitian fermions, which in a result of transition to Hermitian limit ($m_2 \rightarrow 0$, $m_1 \rightarrow m$) coincide with the ordinary particles (see Fig.1 and [2]). But there are the second region where fermions do not subordinate to the ordinary Dirac equations and for them the Hermitian transition is absent. It is easy to see that (6) can be used for new restrictions of mass parameters. Really, if we take into account inequality between arithmetic and geometrical averages of two positive numbers we have

$$m^2 + m_2^2 \geq 2\sqrt{m^2 m_2^2}. \quad (13)$$

On the foundation of this inequality it can formulate five important Remarks.

Remark 1. *The pseudo-Hermitian approach with γ_5 -mass extension contains restriction of mass parameter beside (6). Indeed from (13) follows,*

that the sign of equality takes place when $m = m_2$. If we use parameter α then from (13)

$$1 + \sinh^2 \alpha = 2 \sinh \alpha. \quad (14)$$

From (14) we can also see that in this point $\sinh \alpha = 1$ and solving this equation we can find the value:

$$\alpha_0 = 0.881$$

Remark 2. *Maximal value of mass particle $m = M$ is achieved in the point $m_2 = M$ $m_1 = \sqrt{2}M$. The proof of this fact can be confirmed by the way defining the mass of the Maximon. Indeed, under α_0 we have the equality $m \sinh \alpha_0 = M = m_2$. and also for m_1 we have $m_1 = m \cosh \alpha_0 = \sqrt{2}M$.*

Remark 3. *The particle with the maximal mass (Maximon) is the pseudo-Hermitian fermion. Using Remark 2. for Maximon we can obtain expression*

$$M_{Maximon} = \sqrt{2}M + \gamma_5 M. \quad (15)$$

This phenomenon may be given a very simple physical interpretation. This means that particles with the maximal mass (Maximons) $m = M$ are non-Hermitian (pseudo-Hermitian) fermions.

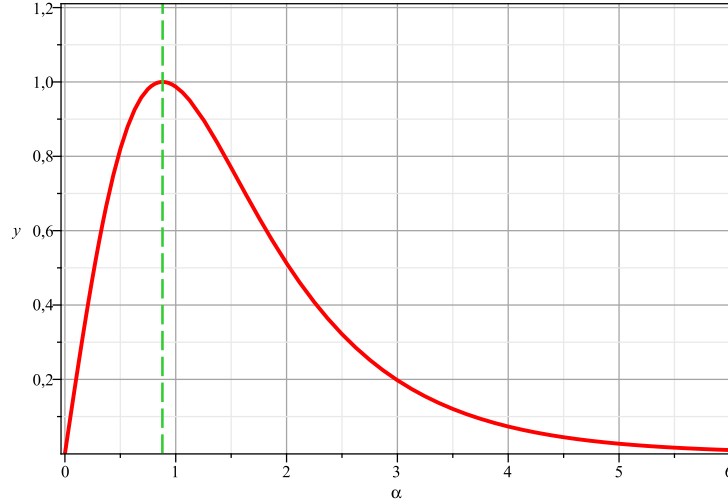


Figure 2: Dependence of $y(\alpha) = \tilde{m} = m/M$ on the parameter α

At the Fig.2 we can see explicit behavior of the reduced mass distribution of $y(\alpha) = m/M$, depending on parameter α . From this picture follows that

the curve, corresponding mass of the considered particles, has a maximum. This the maximal value of $y(\alpha) = m/M = 1$ corresponds to $\alpha_0 = 0.881$ that as already noted correspond to Maximon. Till to this value we are dealing with fermions, which have Hermitian limit when $M \rightarrow \infty$ (or $m_2 \rightarrow 0$). But after the value $\alpha_0 = 0.881$ is achieved we once more deal with decreasing mass of particles. However in this region already no the possibility with the help of the limiting transition to obtain Hermitian mass. Thus in this region particles exist, which in principal differ from the particles of the SM (exotic particles).

Remark 4. This particles (exotic particles) exist thanks to the presence of a maximal value of mass parameter, because the Hermitian limit for them is absent. If one can detect their it means that limiting mass fermions exist and our world is pseudo-Hermitian.

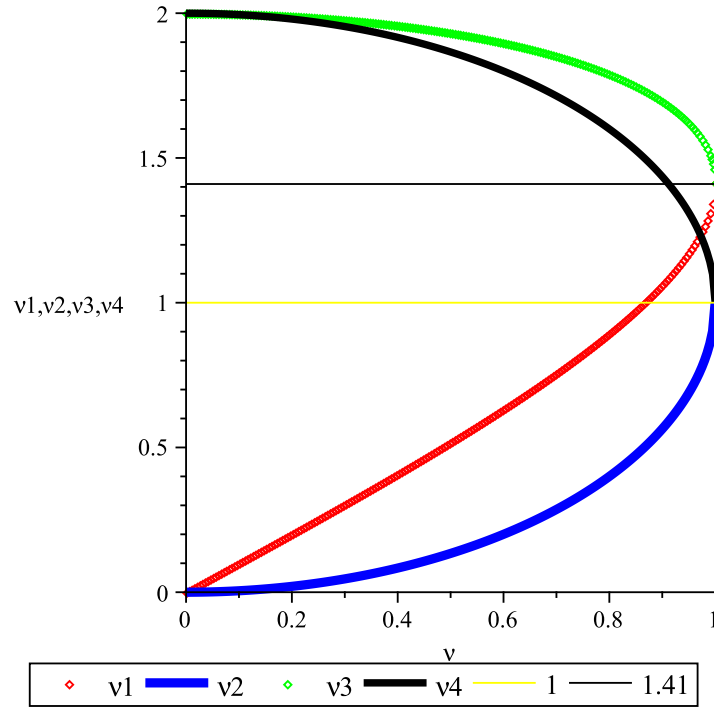


Figure 3: Values of the parameters $\nu_1 = m_1^-/M, \nu_2 = m_2^-/M, \nu_3 = m_1^+/M, \nu_4 = m_2^+/M$ as functions of $\nu = m/M$

Remark 5. And vice versa if the restriction of mass spectrum of elementary particles does not exist then exotic particles can not arise in

Nature. And it is very important because restriction of mass in SM is absent then experimental verification can be start from the most biggest values of maximal mass. In particular, it may be the Planck mass $M = 10^{19} GeV$.

It is very interesting that the early such particles had discovered in geometrical approach to the construction of QFT with fundamental mass [9]-[11]. We believe that exact solutions of modified Dirac-Pauli equations which were obtained by us for the pseudo-Hermitian neutrinos can let valuable information to detect the presence of "exotic particles". This is indicated also increase the effects proportionally $\sim M/m_\nu$ associated with unusual properties of "exotic neutrinos", interacting with the magnetic fields.

At Fig.3 we can see values of different branches of ν_1^\pm and ν_2^\pm as a function of normalized physical parameter ν (see also (4)). The existence the domain of the \mathcal{PT} -symmetry is $0 \leq \nu \leq 1$. For these values of the parameters ν_1 and ν_2 , the modified Dirac equation with the maximum mass describes the propagation of particles with real masses. But the lower branches ν_1^- , ν_2^- correspond to ordinary particles and upper lines ν_1^+ , ν_2^+ define the exotic partners.

3 Modified model for the study of non-Hermitian mass parameters in intensive magnetic fields

In this section, we shell want touch upon question of describing the motion of Dirac particles, if their own magnetic moment is different from the Bohr magneton. As it was shown by Schwinger [16] the equation of Dirac particles in the external electromagnetic field A^{ext} taking into account the radiative corrections may be represented in the form:

$$(\mathcal{P}\gamma - m) \Psi(x) - \int \mathcal{M}(x, y|A^{ext}) \Psi(y) dy = 0, \quad (16)$$

where $\mathcal{M}(x, y|A^{ext})$ is the mass operator of the fermion in the external field and $\mathcal{P}_\mu = p_\mu - A^{ext}_\mu$. From equation (16) by means of expansion of the mass operator in a series of according to eA^{ext} with precision not over then linear field terms one can obtain the modified equation. This equation preserves the relativistic covariance and consistent with the phenomenological equation of Pauli obtained in his early papers (see for example [17]).

Now let us consider the model of massive fermions with γ_5 -extension of mass $m \rightarrow m_1 + \gamma_5 m_2$ taking into account the interaction of their charges and AMM with the electromagnetic field $F_{\mu\nu}$:

$$\left(\gamma^\mu \mathcal{P}_\mu - m_1 - \gamma_5 m_2 - \frac{\Delta\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} \right) \tilde{\Psi}(x) = 0, \quad (17)$$

where $\Delta\mu = (\mu - \mu_0) = \mu_0(g - 2)/2$. Here μ - magnetic moment of a fermion, g - fermion gyromagnetic factor, $\mu_0 = |e|/2m$ - the Bohr magneton, $\sigma^{\mu\nu} = i/2(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$. Thus phenomenological constant $\Delta\mu$, which was introduced by Pauli, is part of the equation and gets the interpretation with the point of view QFT.

The Hamiltonian form of (17) in the homogenies magnetic field is the following

$$i \frac{\partial}{\partial t} \tilde{\Psi}(r, t) = H_{\Delta\mu} \tilde{\Psi}(r, t), \quad (18)$$

where

$$H_{\Delta\mu} = \vec{\alpha} \vec{\mathcal{P}} + \beta(m_1 + \gamma_5 m_2) + \Delta\mu \beta(\vec{\sigma} \mathbf{H}). \quad (19)$$

For example, given the quantum electrodynamic contribution in AMM of an electron with accuracy up to e^2 order we have $\Delta\mu = \frac{\alpha}{2\pi} \mu_0$, where $\alpha = e^2 = 1/137$ - the fine-structure constant and we still believe that the potential of an external field satisfies to the free Maxwell equations.

It should be noted that now the operator projection of the fermion spin at the direction of its movement - $\vec{\sigma} \vec{\mathcal{P}}$ is not commute with the Hamiltonian (19) and hence it is not the integral of motion. The operator μ_3 commuting with this Hamiltonian is the operator of polarization which can be represented in the form of the third component of the polarization tensor is oriented along the direction of the magnetic field [17]

$$\mu_3 = m_1 \sigma_3 + \rho_2 [\vec{\sigma} \vec{\mathcal{P}}]_3 \quad (20)$$

where matrices

$$\sigma_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad \rho_2 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}.$$

Subjecting the wave function $\tilde{\psi}$ to requirement to be eigenfunction of the

operator polarization μ_3 and Hamilton operator (19) we can obtain:

$$\mu_3 \tilde{\psi} = \zeta k \tilde{\psi}, \quad \mu_3 = \begin{pmatrix} m_1 & 0 & 0 & \mathcal{P}_1 - i\mathcal{P}_2 \\ 0 & -m_1 & -\mathcal{P}_1 - i\mathcal{P}_2 & 0 \\ 0 & -\mathcal{P}_1 + i\mathcal{P}_2 & m_1 & 0 \\ \mathcal{P}_1 + i\mathcal{P}_2 & 0 & 0 & -m_1 \end{pmatrix}, \quad (21)$$

where $\zeta = \pm 1$ are characterized the projection of fermion spin at the direction of the magnetic field, and

$$H_{\Delta\mu} \tilde{\psi} = E \tilde{\psi},$$

$$H_{\Delta\mu} = \begin{pmatrix} m_1 + H\Delta\mu & 0 & \mathcal{P}_3 - m_2 & \mathcal{P}_1 - i\mathcal{P}_2 \\ 0 & m_1 - H\Delta\mu & \mathcal{P}_1 + i\mathcal{P}_2 & -m_2 - \mathcal{P}_3 \\ m_2 + \mathcal{P}_3 & \mathcal{P}_1 - i\mathcal{P}_2 & -m_1 - H\Delta\mu & 0 \\ \mathcal{P}_1 + i\mathcal{P}_2 & m_2 - \mathcal{P}_3 & 0 & H\Delta\mu - m_1 \end{pmatrix}. \quad (22)$$

4 Exact solutions of Dirac-Pauli equations in the intensive uniform magnetic field

Performing calculations in many ways reminiscent of similar calculations carried out in the ordinary model in the magnetic field [17] - [20], in a result, for modified Dirac-Pauli equation one can find *the exact solution for energy spectrum* [12],[13]:

$$E(\zeta, p_3, 2\gamma n, H) = \sqrt{p_3^2 - m_2^2 + \left[\sqrt{m_1^2 + 2\gamma n} + \zeta \Delta\mu H \right]^2} \quad (23)$$

and for eigenvalues of the operator polarization μ_3 we can write in the form

$$k = \sqrt{m_1^2 + 2\gamma n}. \quad (24)$$

From (23) it follows that in the field where \mathcal{PT} symmetry is unbroken $m \leq M$, all energy levels are real for the case of spin orientation along the magnetic field direction $\zeta = +1$.

However, in the opposite case $\zeta = -1$ we have the imaginary part from the ground state of fermion $n = 0$ and other low energy levels, see on Fig.4. For the cases of increasing parameter $\Delta\mu H = 0.2$ we can watch overlapping of different levels.

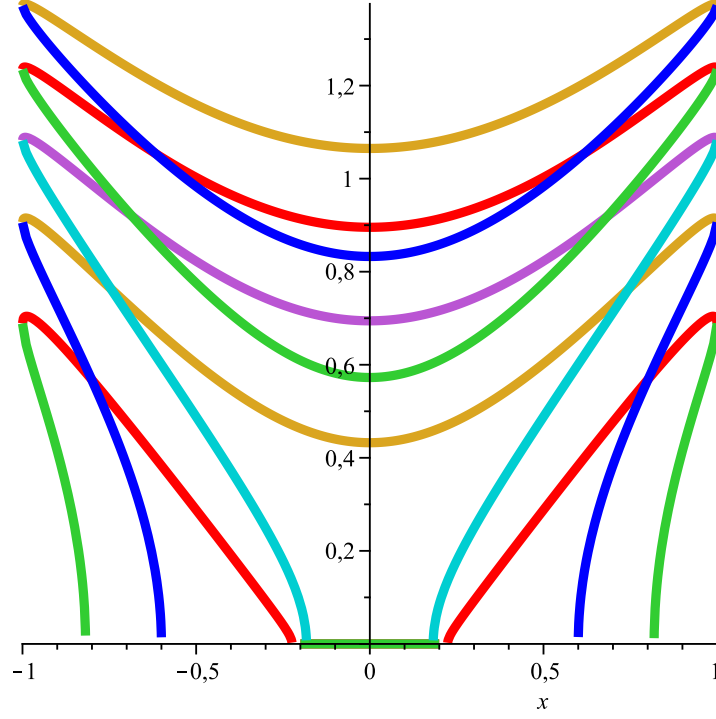


Figure 4: Dependence of $E(-1, 0, 0.4n, 0.1)$ on the parameter $x = m/M$ for the cases $n = 0, 1, 2, 3, 4$ and $\Delta\mu H = 0.1$.

It is easy to see that in the case $\Delta\mu = 0$ from (23) one can obtain the ordinary expression for energy of charged particle in the magnetic field (*Landau levels*). Besides it should be emphasized that from the expression (23), in the Hermitian limit putting $m_2 = 0$ and $m_1 = m$ one can obtain:

$$E(\zeta, p_3, 2\gamma n, H) = \sqrt{p_3^2 + \left[\sqrt{m^2 + 2\gamma n} + \zeta \Delta\mu H \right]^2}. \quad (25)$$

Note that in the paper [21] was previously obtained result analogical to (25) by means of using of the Hermitian Dirac-Pauli approach. Direct comparison of formula (25) with the result [21] shows their coincidence in the Hermitian limit $M \rightarrow \infty$. It is easy to see that the expression (23) contains dependence on parameters m_1 and m_2 separately, which are not combined into *a mass of particles*, that essentially differs from the examples which were considered early [2]-[4].

Thus, here the calculation of interaction AMM of fermions with the magnetic field allow to put the question about the possibility of experimental studies of the non-Hermitian effects of γ_5 -extensions of a fermion mass. Thus, taking into account the expressions (23) we obtain that the energetic spectrum is expressed through the fermion mass m and the value of the maximal mass M . Thus, taking into account that the interaction AMM with magnetic field removes the degeneracy on spin variable, we can obtain the energy of the ground state ($\zeta = -1$) in the form which dependence is represented at Fig.4.

Thus, it is shown that the main progress, is obtained by us in the algebraic way of the construction of the fermion model with γ_5 -mass term is consists of describing of the new energetic scale, which is defined by the parameter $M = m_1^2/2m_2$. This value on the scale of the masses is a point of transition from the ordinary particles $m_2 < M$ to exotic $m_2 > M$. Furthermore, description of the exotic fermions in the algebraic approach are turned out essentially the same as in the model with a maximal mass, which was investigated by V.G.Kadyshevsky with colleagues on the basis of geometrical approach [9]-[11].

It should be noted that the formula (23) is a valid not only for charged fermions, but and for the neutral particles possessing AMM. In this case one must simply replace the value of quantized transverse momentum of a charged particle in a magnetic field on the ordinary value $2\gamma n \rightarrow p_1^2 + p_2^2$. Thus, for the case of ultra cold polarized ordinary electronic neutrino with precision not over then linear field terms we can write

$$E(-1, 0, 0, H, M \rightarrow \infty) = m_{\nu_e} \sqrt{1 - \frac{\mu_{\nu_e}}{\mu_0} \frac{H}{H_c}}. \quad (26)$$

However, in the case of exotic electronic neutrino \tilde{m}_{ν_e} we have

$$E(-1, 0, 0, H, \tilde{m}_{\nu_e}/M) = \tilde{m}_{\nu_e} \sqrt{1 - \frac{\mu_{\nu_e}}{\mu_0} \frac{2MH}{\tilde{m}_{\nu_e} H_c}}. \quad (27)$$

It is well known [22],[23] that in the minimally extended SM the one-loop radiative correction generates neutrino magnetic moment which is proportional to the neutrino mass

$$\mu_{\nu_e} = \frac{3}{8\sqrt{2}\pi^2} |e| G_F m_{\nu_e} = (3 \cdot 10^{-19}) \mu_0 \left(\frac{m_{\nu_e}}{1\text{eV}} \right), \quad (28)$$

where G_F -Fermi coupling constant and μ_0 is Bohr magneton. However note that the best laboratory upper limit on a neutrino magnetic moment, $\mu \leq 2.910^{-11}\mu_0$, has been obtained by the GEMMA collaboration [24], and the best astrophysical limit is $\mu \leq 3 \cdot 10^{-12}\mu_0$.

Existence masses of neutrino and mixing implies that neutrinos have magnetic moments. In last time one can often meet with an overviews of electromagnetic properties neutrino, (see, for example, [25]). But as it was noted up in this paper " now there is no positive experimental indication in favor existence electromagnetic properties of neutrinos". With it really is hard not to agree because the interactions of ordinary neutrinos with the electromagnetic fields are extremely weak. However if one to suggest using the "exotic neutrinos" the interaction with magnetic field may be really significantly increased thanks to the coefficient which be equal to the ratio of maximal mass and mass of neutrinos $k = M/m_\nu$ [12],[13]. Such experiments in our opinion may be very fruitful for creation the new physics beyond the SM. Perhaps that this effects indeed can be observed in terrestrial experiments.

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